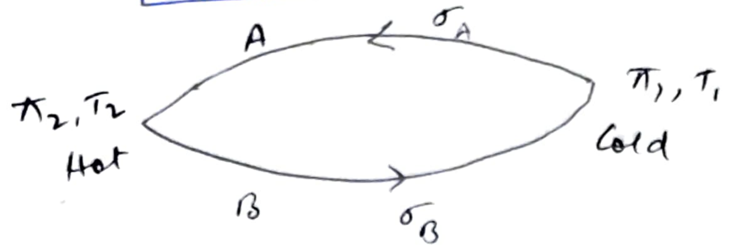
Current Electricity

$$\boxed{\text{Prove: } \pi = T\frac{dE}{dT}} \quad ; \quad \frac{dE}{dT} = \text{Thermoelectric Power}$$

π = Peltier Coefficient

T = Absolute temperature



Consider a thermocouple of two metals A and B with their junctions at temperature T_2 and T_1. The peltier Coefficient at temperature T_1 is π_1 and at temperature T_2 is π_2; and σ_A and σ_B are the Thomson Coefficients of A and B. If a Current I ampere passes for t second then,

Energy absorbed due to Peltier effect at the hot junction $= \pi_2 I t$

Energy absorbed due to Peltier's effect at the Cold junction

$= -\pi_1 I t$ (−ve sign, because energy is evolved)

Energy absorbed in metal A due to Thomson effect

$$= \left(\int_{T_1}^{T_2} \sigma_A \, dT\right) I t$$

Energy absorbed in metal B due to Thomson effect

$$= \left(-\int_{T_1}^{T_2} \sigma_B \, dT\right) I t$$

The −ve sign before the integral indicates that Current flows from higher to lower temperature. The values of σ_A and σ_B are taken as positive or negative depending on whether the metal A or B shows positive or negative.

Thomson effect.

$\therefore$ Total gain in energy for the Complete Circuit

$$= \left[(\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT \right] I t \quad — ①$$

The total thermoemf produced in the circuit = E volt

$\therefore$ Energy produced = $E I t$ — ②

$\therefore$ Equating ① and ②

$$E I t = \left[(\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT \right] I t$$

$$E = (\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT$$

If the two junctions are at a difference of temperature dT and the difference in Peltier's Coefficient is $d\pi$, then

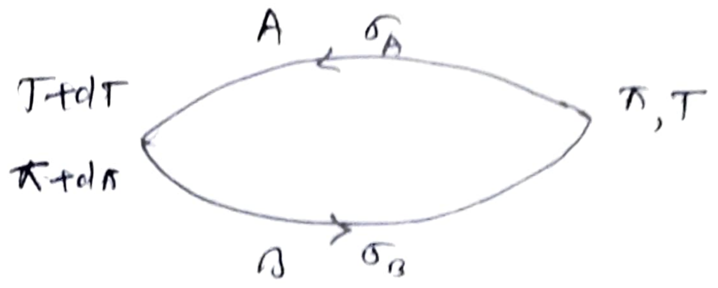
$$\boxed{dE = d\pi + (\sigma_A - \sigma_B)\, dT}$$

According the second law of thermodynamics, in complete reversible process.

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

Q_1 = Heat absorbed at temp T_1

Q_2 = Heat rejected at temp. T_2



The two junction are at temperature T and T+dT and a current of I amperature flows for t second.

Energy absorbed at the hot junction due to Peltier effect

$$= (\pi + d\pi) I t \text{ Joule} = \frac{(\pi + d\pi) I t}{J} \text{ Cal}$$

Energy evolved at the cold junction due to Peltier effect $= \pi I t$ joule $= \frac{\pi I t}{J}$ Cal.

Total energy absorbed due to Thomson effect

$$= \left[(\sigma_A - \sigma_B)\, dT\right] I t \text{ Joule}$$

$$= \frac{\left[(\sigma_A - \sigma_B)\, dT\right] I t}{J} \text{ Cal.}$$

$\therefore$ Applying the second law of thermodynamics.

$$\frac{(\pi + d\pi) I t}{J\,(T+dT)} + \frac{\left[(\sigma_A - \sigma_B)\, dT\right] I t}{J(T)} = \frac{\pi I t}{J(T)}$$

$$= \frac{\pi + d\pi}{T+dT} - \frac{\pi}{T} + \frac{(\sigma_A - \sigma_B)\, dT}{T} = 0$$

$$\frac{\pi T + T d\pi - \pi T - \pi dT}{T(T+dT)} + \frac{(\sigma_A - \sigma_B)\, dT}{T} = 0$$

$$[T(T+dT) = T^2 \text{ (approx.)}]$$

$$\therefore \frac{T d\pi}{T^2} - \frac{\pi dT}{T^2} + \frac{(\sigma_A - \sigma_B) dT}{T} = 0$$

$$d\pi - \frac{\pi dT}{T} + (\sigma_A - \sigma_B) dT = 0$$

$$\therefore d\pi + (\sigma_A - \sigma_B) dT = \frac{\pi dT}{T}$$

But $d\pi + (\sigma_A - \sigma_B) dT = dE$

$$\therefore dE = \frac{\pi dT}{T}$$

$$\boxed{\pi = T\frac{dE}{dT}}$$

where $\frac{dE}{dT} = P$ = Thermoelectric Power

Hence, Peltier's Coefficient at a given junction is the product of the absolute temperature and the rate of change of the thermo emf with respect to temperature.